A Network Analysis of the Volatility of High-Dimensional Financial Series

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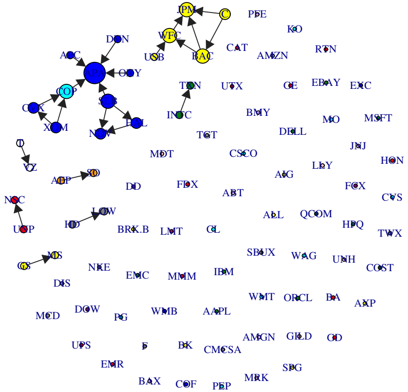
9 December 2020

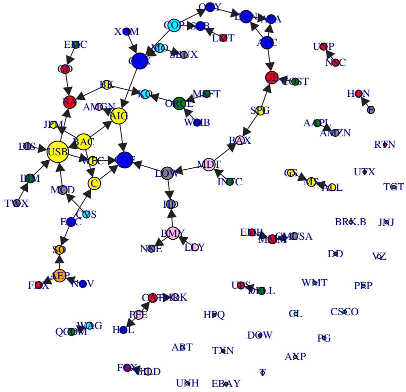
In a nutshell - aim and methods

1. We study the network of S&P100 stocks' volatilities;
2. for financial data no network structure pre-exists the observations
   $\Rightarrow$ we consider Long Run Variance Decomposition Networks;
   Diebold and Yılmaz, 2014
3. large dimensional system of time series poses difficulties in estimation
   $\Rightarrow$ we use factor models and lasso-type regressions as solutions;
4. financial shocks have an economic meaning
   $\Rightarrow$ we identify shocks by means of recursive identification scheme;
5. higher connectedness means higher uncertainty
   $\Rightarrow$ we extract volatilities as measures of fear or lack of confidence.

In a nutshell - results



2000-2013



2007-2008
Great Financial Crisis

Data

A panel of stock daily returns

$$\boldsymbol{r} = \{r_{it} | i = 1, \ldots, n, \; t = 1, \ldots, T\}$$

- from Standard & Poor's 100 index;
- $n = 90$ assets from 10 sectors: Consumer Discretionary, Consumer Staples, Energy, Financial, Health Care, Industrials, Information Technology, Materials, Telecommunication Services, Utilities;
- $T = 3457$ days from 3rd January 2000 to 30th September 2013;
- from returns we extract volatilities which are unobserved;
- we are in a large n, T setting.

Long Run Variance Decomposition Network (LVDN)

- Weighted and directed graph;
- the weight associated with edge (i, j) represents the proportion of h-step ahead forecast error variance of variable i which is accounted for by the innovations in variable j;
- completely characterised by the infinite vector moving average (VMA) representation given by Wold's classical representation theorem;
- for a generic process $\boldsymbol{Y}$ the model reads

$$\boldsymbol{Y}_t = \boldsymbol{D}(L)\boldsymbol{e}_t, \quad \boldsymbol{e}_t \sim w.n.(\mathbf{0}, \mathbf{I})$$

where
$\boldsymbol{e}_t$ are orthonormal shocks with an economic meaning
$\boldsymbol{D}(L)$ are impulse response functions (IRF) which give the network

Long-Run Variance Decomposition Network (LVDN)

- Vertices set $\mathcal{V} = \{1 \dots n\}$;
- edges set
$$\mathcal{E}_{LVDN} = \left\{(i,j) \in \mathcal{V} \times \mathcal{V} | \lim_{h\to\infty} w_{ij}^h \neq 0\right\}$$
- edges weights
$$w_{ij}^h = 100 \left(\frac{\sum_{k=0}^{h-1} d_{k,ij}^2}{\sum_{\ell=1}^{n} \sum_{k=0}^{h-1} d_{k,i\ell}^2} \right)$$
where $d_{k,ij}$ is entry of $\mathbf{D}_{nk}$ such that $\mathbf{D}_n(L) = \sum_{k=0}^{\infty} \mathbf{D}_{nk} L^k$;
- w_{ij}^h is the proportion of h-step ahead forecast error variance of Z_i which is accounted for by the innovations in Z_j.

Long-Run Variance Decomposition Network (LVDN)

- The operative definition of LVDN requires to fix h;
- the weights are normalised

$$\frac{1}{100}\sum_{j=1}^{n} w_{ij}^{h} = 1, \qquad \frac{1}{100}\sum_{i,j=1}^{n} w_{ij}^{h} = n;$$

- we define the FROM and TO degrees as

$$\delta_i^{FROM} = \sum_{\substack{j=1 \\ j\neq i}}^{n} w_{ij}^{h}, \qquad \delta_j^{TO} = \sum_{\substack{i=1 \\ i\neq j}}^{n} w_{ij}^{h};$$

- a measure of total connectedness is given by

$$\delta^{TOT} = \frac{1}{n}\sum_{i=1}^{n} \delta_i^{FROM} = \frac{1}{n}\sum_{j=1}^{n} \delta_j^{TO}.$$

Goal 1 - estimate a VMA

- Estimate and invert VAR (classical approach);
- in a large dimensional setting $\Rightarrow$ curse of dimensionality;
- two main solutions in time series:
  1. factor models - dense modeling;

     Forni, Hallin, Lippi, Reichlin, 2000, Forni, Hallin, Lippi, Zaffaroni, 2017, Barigozzi and Hallin, 2020
  2. lasso-type penalised regressions - sparse modeling;

     Peng, Wang, Zhou, Zhu, 2009, Kock and Callot, 2015, Barigozzi and Brownlees, 2019
- used to analyse two complementary features of financial markets:
  1. effect of global shocks $\Rightarrow$ pervasive risk, non-diversifiable;

     Ross, 1976, Chamberlain and Rothschild, 1983, Fama and French, 1993
  2. effect of idiosyncratic shocks $\Rightarrow$ systemic risk, limited diversifiability;

     Gabaix, 2011, Acemoglu, Carvalho, Ozdaglar, Tahbaz-Salehi, 2012
- Economic data are more likely to be dense rather than sparse.

  Giannone, Primiceri, Lenza, 2018

Goal 2 - identify the shocks

- Given a VMA any invertible linear transformation of the shocks is a statistically valid representation;
- to attach an economic meaning to the shocks $\Rightarrow$ to identify;
  - assume orthonormality or even independence;
  - recursive identification schemes, i.e. choose shocks' ordering.

Generalised Dynamic Factor Model (GDFM)

Consider a generic $n \times T$ panel of time series $\mathbf{Y}_n$ such that

A1 $\mathbf{Y}_n$ is strongly stationary;

A2 its spectral density $\boldsymbol{\Sigma}_n(\theta)$ exists, is rational, with eigenvalues $\lambda_{j,n}(\theta)$;

A3 there exists a $q < n$ not depending on n such that:

- **a** $\lambda_{q,n}(\theta) \to \infty$ as $n \to \infty$;
- **b** $\lambda_{q+1,n}(\theta) < M < \infty$ for any $n \in \mathbb{N}$.

Generalised Dynamic Factor Model (GDFM)

Under A1-A3

$$\mathbf{Y}_{nt} = \mathbf{X}_{nt} + \mathbf{Z}_{nt} = \mathbf{B}_n(L)\mathbf{u}_t + \mathbf{Z}_{nt} \tag{1}$$

i $\mathbf{u}$ is q-dimensional and $\mathbf{u}_t \sim w.n.(\mathbf{0}, \mathbf{I})$
$\Rightarrow$ global shocks;

ii $\mathbf{B}_n(L)$ is $n \times q$ polynomials with squared summable coefficients
$\Rightarrow$ IRFs to global shocks;

iii q spectral eigenvalues of $\mathbf{X}_n$ diverge as $n \to \infty$;
$\Rightarrow$ strong (auto)correlation among components of $\mathbf{X}_n$;

iv n spectral eigenvalues of $\mathbf{Z}_n$ are bounded for any $n \in \mathbb{N}$;
$\Rightarrow$ weak, but not zero, (auto)correlation among components of $\mathbf{Z}_n$;

v $\mathbf{X}_n$ and $\mathbf{Z}_n$ are mutually orthogonal at every lead and lag.

Notice that

model and assumptions are defined in the limit $n \to \infty$;

under A1-A2, model (1) and A3 are equivalent.

Idiosyncratic component - VMA

The idiosyncratic component admits the Wold decomposition

$$\mathbf{Z}_{nt} = \mathbf{D}_n(L)\mathbf{e}_{nt}$$

vi $\mathbf{e}_n$ is n-dimensional and $\mathbf{e}_{nt} \sim w.n.(\mathbf{0}, \mathbf{I})$
$\Rightarrow$ idiosyncratic shocks;

vii $\mathbf{D}_n(L)$ is $n \times n$ polynomials with squared summable coefficients
$\Rightarrow$ IRFs to idiosyncratic shocks.

Shocks

Two sources of variation:

1. few (q) global shocks, $\mathbf{u}$, with pervasive effect
   due to condition (iii) of diverging eigenvalues;

2. many (n) idiosyncratic shocks, $\mathbf{e}_n$, with limited, but not null, effect
   due to condition (iv) of bounded eigenvalues
   $\Rightarrow$ no sparsity assumption is made.

We first control for the global effects and then we focus on the effect of idiosyncratic shocks measured through the LVDN.

Idiosyncratic component - VAR

To estimate a VMA we assume

A4 $\mathbf{Z}_n$ has the sparse-VAR(p) representation

$$\mathbf{F}_n(L)\mathbf{Z}_{nt} = \mathbf{v}_{nt}, \qquad \mathbf{v}_{nt} \sim w.n.(\mathbf{0}, \mathbf{C}_n^{-1})$$

where $\mathbf{F}_n(L) = \sum_{k=0}^{p} \mathbf{F}_{nk} L^k$ with $\mathbf{F}_{n0} = \mathbf{I}$ and $\det(\mathbf{F}_n(z)) \neq 0$ for any $z \in \mathbb{C}$ such that $|z| \leq 1$, and $\mathbf{C}_n$ has full-rank. Moreover, $\mathbf{F}_{nk}$ and $\mathbf{C}_n$ are sparse matrices.

Notice that

we assume sparsity of VAR and not of VMA for estimation purposes but the argument of condition (iv) of bounded eigenvalues still holds; for convenience in the identification step, we parametrise the covariance matrix of the VAR innovations by means of its inverse $\mathbf{C}_n$

Idiosyncratic component - VAR

As a by-product, we have a Long-Run Granger Causality Network (LGCN)

- Edges set

$$\mathcal{E}_{LGCN} = \left\{ (i,j) \in \mathcal{V} \times \mathcal{V} | \sum_{k=0}^{p} f_{kij} \neq 0 \right\}$$

- it captures the leading/lagging conditional linear dependencies;

  Dahlhaus and Eichler, 2003, Eichler, 2007, Barigozzi and Brownlees, 2019
- under A4 the LGCN is likely to be sparse but the LVDN is not necessarily sparse;
- the economic interpretation of the LGCN is not as straightforward as that of the LVDN, and the LGCN therefore is of lesser interest for the analysis of financial systems and we consider it just as a tool to derive the LVDN;
- cfr. with traditional macroeconomic analysis where IRF, i.e. VMA coefficients, rather than VAR ones, are the object of interest for policy makers.

Identification

From the VAR and VMA of $\mathbf{Z}_n$ we have

$$\mathbf{D}_n(L) = (\mathbf{F}_n(L))^{-1}\mathbf{R}_n$$

where $\mathbf{R}_n$ is such that it makes the shocks $\mathbf{R}_n^{-1}\mathbf{v}_n = \mathbf{e}_n$ orthonormal.

- choosing $\mathbf{R}_n$ is equivalent to identifying the shocks;
- choose $\mathbf{R}_n$ as the lower triangular matrix such that

  $$\text{Cov}(\mathbf{v}_n) = \mathbf{C}_n^{-1} = \mathbf{R}_n\mathbf{R}_n'$$

  then

  $$\text{Cov}(\mathbf{e}_n) = \mathbf{R}_n^{-1}\text{Cov}(\mathbf{v}_n)\mathbf{R}_n^{-1'} = \mathbf{R}_n^{-1}\mathbf{R}_n\mathbf{R}_n'\mathbf{R}_n^{-1'} = \mathbf{I}$$

- but this choice depends on the ordering of the shocks, a given order of shocks defines which component we choose to hit first.

Identification

We use $\mathbf{v}_n$'s partial correlation structure

- the partial correlation between v_i and v_j is

$$\rho^{ij} = \frac{-[\mathbf{C}_n]_{ij}}{\sqrt{[\mathbf{C}_n]_{ii}[\mathbf{C}_n]_{jj}}}$$

- associated is the Partial Correlation Network (PCN), with edges set

$$\mathcal{E}_{PCN} = \left\{(i,j) \in \mathcal{V} \times \mathcal{V} |\, \rho^{ij} \neq 0\right\}$$

- by A4, the PCN is a sparse network;

  Peng, Wang, Zhou, Zhu, 2009, Barigozzi and Brownlees, 2019

- order shocks by decreasing eigenvector centrality in the PCN;
- we are considering the case in which the most contemporaneously interconnected node is firstly affected by an unexpected shock, and then, by means of the subsequent impulse response analysis, we study the propagation of such shock through the whole system.

Returns vs. Volatilities

- returns are observed but volatilities are unobserved
  $\Rightarrow$ factor model for returns to extract volatilities in a multivariate way
  Barigozzi and Hallin, 2016, 2017, 2020
- in a univariate setting a generic model for volatility reads as

$$a(L)r_t = \eta_t, \quad \eta_t \sim w.n.(0, \sigma^2), \quad \eta_t^2 = f(\eta_{t-1} \dots \eta_0)$$

- for example stochastic volatility models where $\log \eta^2$ is an AR(1)

$$\log \eta_t^2 = c + a \log \eta_{t-1}^2 + \nu_t.$$

Returns vs. Volatilities

- in the large n case we use an AR of the GDFM

  Forni, Hallin, Lippi, Zaffaroni, 2017

$$\mathcal{A}_n(L)\boldsymbol{r}_{nt} = \boldsymbol{\eta}_{nt} + \boldsymbol{\xi}_{nt}$$

  - $\boldsymbol{\eta}_{nt} = \mathcal{H}_n\mathbf{u}_t$ with $\mathcal{H}_n$ full-rank and $n \times q$ and $\mathbf{u}_t \sim w.n.(\mathbf{0}, \mathbf{I})$
    $\Rightarrow$ global shocks to returns, that is market shocks;
  - $\mathcal{A}_n(L)$ is block-diagonal with blocks of size $q+1$;
  - $\boldsymbol{\xi}_n$ has bounded spectral eigenvalues, is idiosyncratic such that it has the sparse VAR representation

$$\mathcal{F}_n(L)\boldsymbol{\xi}_{nt} = \boldsymbol{v}_{nt}$$

Returns vs. Volatilities

- centred log-volatilities are defined as

$$\boldsymbol{\sigma}_{nt} = \log(\boldsymbol{\eta}_{nt} + \boldsymbol{v}_{nt})^2 - \mathrm{E}[\log(\boldsymbol{\eta}_{nt} + \boldsymbol{v}_{nt})^2]$$

- we assume a GDFM also for log-volatilities

$$\boldsymbol{\sigma}_{nt} = \boldsymbol{\chi}_{\sigma,nt} + \boldsymbol{\xi}_{\sigma,nt}$$
$$\mathbf{A}_{\sigma,n}(L)\boldsymbol{\chi}_{\sigma,nt} = \mathbf{H}_{n,\sigma}\boldsymbol{\varepsilon}_t$$

  - ε is Q-dimensional and $\varepsilon_t \sim w.n.(\mathbf{0}, \mathbf{I})$
    $\Rightarrow$ global shocks to volatilities, that is risk market shocks;
  - $\mathbf{H}_{n,\sigma}$ is $n \times Q$ and $\mathbf{H}'_{n,\sigma}\mathbf{H}_{n,\sigma} = n\mathbf{I}$
    $\Rightarrow$ global shocks are pervasive;
  - $\mathbf{A}_n(L)$ is block-diagonal with blocks of size $Q+1$;
  - $\boldsymbol{\xi}_{\sigma,n}$ has bounded spectral eigenvalues, is idiosyncratic such that it has the sparse VAR representation

$$\mathbf{F}_n(L)\boldsymbol{\xi}_{\sigma,nt} = \boldsymbol{\nu}_{nt}$$

Estimation in one slide

- GDFM estimation is based on the following "tools" (details omitted): Spectral density matrix, dynamic PCA, autocovariances by inverse Fourier transform, Yule Walker equations; Forni, Hallin, Lippi, Zaffaroni, 2017
- run GDFM estimation twice: Barigozzi and Hallin, 2016, 2017, 2020
  1. on returns to obtain shocks $\mathbf{u}$ and $\mathbf{v}_n$ for computing log-volatilities;
  2. on volatilities to obtain the idiosyncratic component $\boldsymbol{\xi}_{\sigma,n}$;
- estimate a sparse VAR on $\boldsymbol{\xi}_{\sigma,n}$, some options are
  1. elastic net Zou and Hastie, 2005
  2. group lasso Yuan and Lin, 2006, Nicholson, Bien, Matteson, 2014, Gelper, Wilms, Croux, 2016
  3. adaptive lasso Zou, 2006, Kock and Callot, 2015, Barigozzi and Brownlees, 2019
  4. other penalties are also possible Hsu, Hung, Chang, 2008, Abegaz and Wit, 2013
- Consistency, as $n, T \to \infty$:
  1. for the double GDFM estimator—Barigozzi and Hallin, 2020;
  2. for the adaptive lasso on a large VAR—Barigozzi and Brownlees, 2019.

The effect of global shocks

The VMA for common volatilities

$$\begin{aligned}\boldsymbol{\chi}_{\sigma,nt} &= (\mathbf{A}_n(L))^{-1}\mathbf{H}_{\sigma,n}\mathbf{K}\boldsymbol{\varepsilon}_t \\ &= \mathbf{B}_n(L)\boldsymbol{\varepsilon}_t\end{aligned}$$

- $\boldsymbol{\varepsilon}_t \sim w.n.(\mathbf{0}, \mathbf{I})$ by construction
  $\Rightarrow$ global shocks to volatilities;
- $\mathbf{K}$ for identification (easy since there are few shocks);
- truncate $(\mathbf{A}_n(L))^{-1}\mathbf{H}_{\sigma,n}\mathbf{K}$ at lag $h = 20$ (one month);
- from the entries of $\mathbf{B}_n(L)$ we can compute percentages of h-step ahead forecast error variances due to the global shocks.

The effect of idiosyncratic shocks (LVDN)

The VMA for idiosyncratic volatilities

$$\begin{aligned}\xi_{\sigma,nt} &= (\mathbf{F}_n(L))^{-1}\boldsymbol{\nu}_{nt} \\ &= (\mathbf{F}_n(L))^{-1}\mathbf{R}_n\mathbf{R}_n^{-1}\boldsymbol{\nu}_{nt} \\ &= \mathbf{D}_n(L)\mathbf{e}_{nt}\end{aligned}$$

- $\mathbf{e}_{nt} \sim w.n.(\mathbf{0}, \mathbf{I})$ by construction
  $\Rightarrow$ idiosyncratic shocks to volatilities;
- $\mathbf{R}_n$ identified using centrality in the PCN of $\boldsymbol{\nu}_{nt}$;
- truncate $(\mathbf{F}_n(L))^{-1}\mathbf{R}_n$ at lag $h = 20$ (one month);
- LVDN weights are given by the entries of $\mathbf{D}_n(L)$
  $\Rightarrow$ weighted directed non-sparse network;
- LVDN can be made sparse by some thresholding method.

Data

- Adjusted daily closing prices p_{it};
- 90 daily stock returns from S&P 100 index $r_{it} = 100\Delta \log p_{it}$;
- 10 sectors: Consumer Discretionary, Consumer Staples, Energy, Financial, Health Care, Industrials, Information Technology, Materials, Telecommunication Services, Utilities;
- two periods 2000-2013 and 2007-2008;
- from returns we extract log-volatilities;
- data are not standardised.

Number of factors

- look at the behaviour of the spectral eigenvalues;

  Hallin and Liška, 2007
- one global shock in returns $q = 1$;
- one global shock in volatilities $Q = 1$;
- in both cases the global shock explains about 40% of total variation

$$EV = \frac{\int_{-\pi}^{\pi} \lambda_{1,n}(\theta)\mathrm{d}\theta}{\sum_{i=1}^{n} \int_{-\pi}^{\pi} \lambda_{i,n}(\theta)\mathrm{d}\theta} \simeq 0.4$$

- the idiosyncratic shocks account for about 60% of total variation.

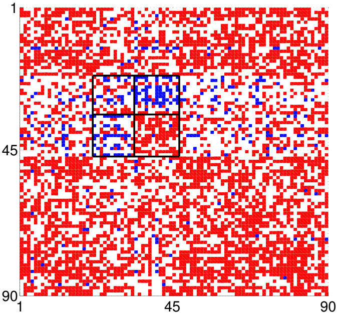
Effects of global volatility shocks

Sector	2000-2013	2007-2008
Cons. Disc.	8.87	8.82
Cons. Stap.	10.54	10.14
Energy	11.61	18.44
Financial	11.89	14.40
Health Care	9.38	8.01
Industrials	8.50	7.97
Inf. Tech.	10.00	6.94
Materials	8.35	9.79
Telecom. Serv.	10.07	8.04
Utilities	10.82	7.47
Total	100	100

Percentages of 20-step ahead forecast error variances due to the global shock.

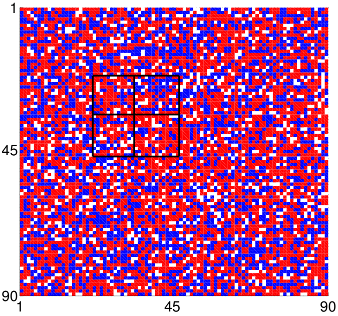
Sparse VAR for idiosyncratic component

VAR order selected: $p = 5$ - Elastic net



2000-2013

density = 0.53



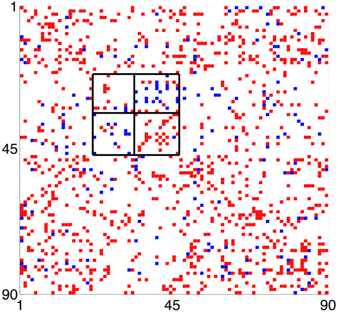
2007-2008

density = 0.86

negative weights in blue, positive weights in red.

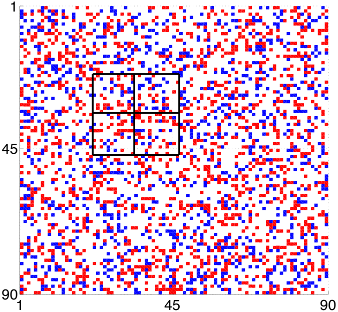
Sparse VAR for idiosyncratic component

VAR order selected: $p = 5$ - Group lasso



2000-2013

density = 0.14

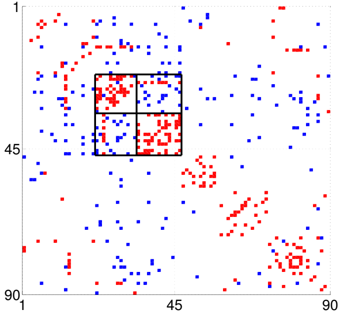


2007-2008

density = 0.32

negative weights in blue, positive weights in red.

PCN for idiosyncratic innovations



2000-2013 — density = 0.06

2007-2008 — density = 0.24

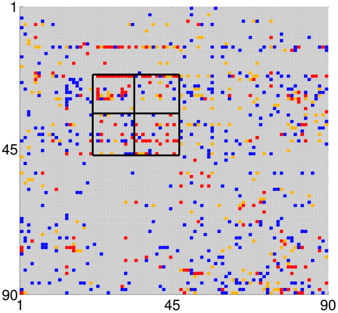
negative weights in blue, positive weights in red.

PCN for idiosyncratic innovations

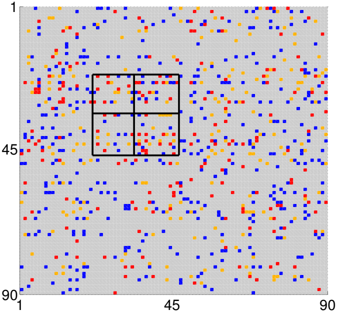
2000-2013	2007-2008
JPM JP Morgan Chase & Co.	BAC Bank of America Corp.
C Citigroup Inc.	USB US Bancorp
BAC Bank of America Corp.	JPM JP Morgan Chase & Co.
APA Apache Corp.	MS Morgan Stanley
WFC Wells Fargo	WFC Wells Fargo
COP Conoco Phillips	DVN Devon Energy
OXY Occidental Petroleum Corp.	GS Goldman Sachs
DVN Devon Energy	AXP American Express Inc.
SLB Schlumberger	COF Capital One Financial Corp.
MS Morgan Stanley	UNH United Health Group Inc.

Eigenvector centrality in the PCN.

LVDN



2000-2013



2007-2008

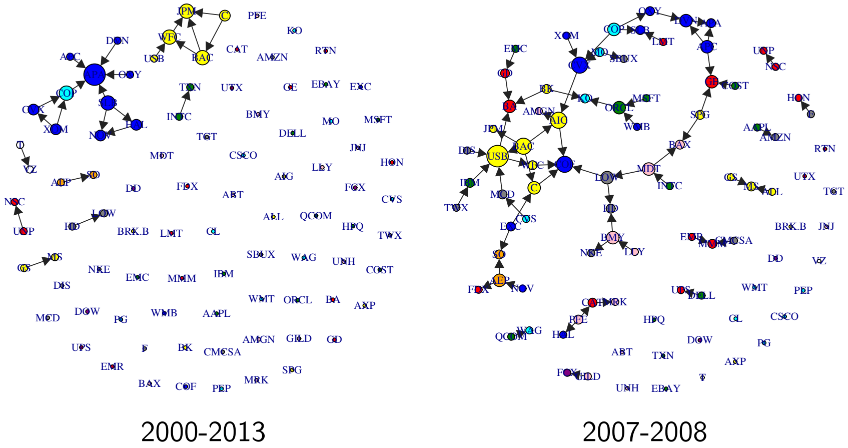
weights below the 95^{th} percentile in grey, between the 95^{th} and 97.5^{th} percentiles in blue, between the 97.5^{th} and 99^{th} percentiles in yellow, and above the 99^{th} percentile in red.

LVDN

percentiles	50^{th}	90^{th}	95^{th}	97.5^{th}	99^{th}	max
2000-2013	0.02	0.13	0.20	0.29	0.48	4.29
2007-2008	0.17	0.71	1.00	1.28	1.76	4.53

Selected percentiles of LVDN weights.

LVDN - sparse



Thresholded LVDN.

LVDN - centrality

2000-2013	2007-2008
BAC Bank of America Corp.	USB US Bancorp
JPM JP Morgan Chase & Co.	BAC Bank of America Corp.
WFC Wells Fargo	COF Capital One Financial Corp.
C Citigroup Inc.	AIG American International Group Inc.
USB US Bancorp	C Citigroup Inc.
APA Apache Corp.	WFC Wells Fargo
SLB Schlumberger	BA Boeing Co.
COP Conoco Phillips	CVX Chevron

Eigenvector centrality in the LVDN.

LVDN - connectivity

	2000-2013		2007-2008	
Sector	from	to	from	to
Cons. Disc.	4.32	2.37	26.31	26.41
Cons. Stap.	3.98	4.65	27.47	22.65
Energy	5.52	7.92	21.91	33.72
Financial	4.74	6.22	24.42	35.56
Health Care	5.00	2.51	28.06	22.36
Industrials	4.43	3.21	27.26	25.81
Inf. Tech.	5.03	4.89	29.90	19.98
Materials	3.24	4.62	26.86	27.01
Telecom. Serv.	6.50	7.26	27.44	16.52
Utilities	5.15	8.74	29.49	21.54
Total degree	4.73		26.54	

From- and To-degree sectoral averages in LVDN.

Summary

- We determine and quantify the different sources of variation driving a panel of volatilities of S&P100 stocks over the period 2000-2013;
- increased connectivity during Financial Crisis;
- key role of the Financial sector, particularly during the Financial Crisis;
- other sectors such as Energy seem to have an important role too;
- a "factor plus VAR" approach motivated by
  - financial interpretation: global vs. idiosyncratic risk;
  - existence of common factors is at odds with sparsity;
  - data structure as shown by partial spectral coherencies;
- results are robust to
  - other VAR estimations as (i) group lasso, (ii) adaptive lasso;
  - different forecasting horizons h;
  - other identifications strategies as (i) centrality of PCN when signs of correlations are accounted for, (ii) generalised variance decomposition.

Thank you!

Questions?

Partial spectral coherence

It is the analogous of partial correlation but in the frequency domain

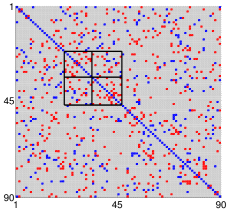
$$PSC_{ij}(\theta) = \frac{-[\mathbf{\Sigma}(\theta)]_{ij}}{\sqrt{[\mathbf{\Sigma}(\theta)]_{ii}[\mathbf{\Sigma}(\theta)]_{jj}}}$$

- directly related to VAR coefficients;

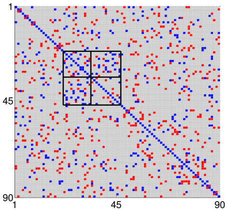
  Davis, Zang, Zheng, 2015

- compare *PSC* of volatilities $\boldsymbol{\sigma}_n$ and idiosyncratic volatilities $\boldsymbol{\xi}_{\sigma;n}$;
- difference between a sparse VAR on $\boldsymbol{\sigma}_n$ vs. sparse VAR on $\boldsymbol{\xi}_{\sigma;n}$.

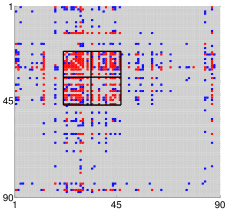
Partial spectral coherence



$PSC_{\sigma_n}(\theta = 0)$



$PSC_{\boldsymbol{\xi}_{\sigma;n}}(\theta = 0)$



$|PSC_{\boldsymbol{\sigma}_n}(\theta = 0) - PSC_{\boldsymbol{\xi}_{\sigma;n}}(\theta = 0)|$

Left and middle panels: weights in absolute values below the 90$^{\text{th}}$ percentile in grey, weights above the 90$^{\text{th}}$ percentile in red, and below the 10$^{\text{th}}$ percentile in blue. Right panel: weights below the 90$^{\text{th}}$ percentile in grey, between the 90$^{\text{th}}$ and 95$^{\text{th}}$ percentiles in blue, and above the 95$^{\text{th}}$ percentile in red.