Is the Carli index flawed? Assessing the case for the new RPIJ

Peter Levell

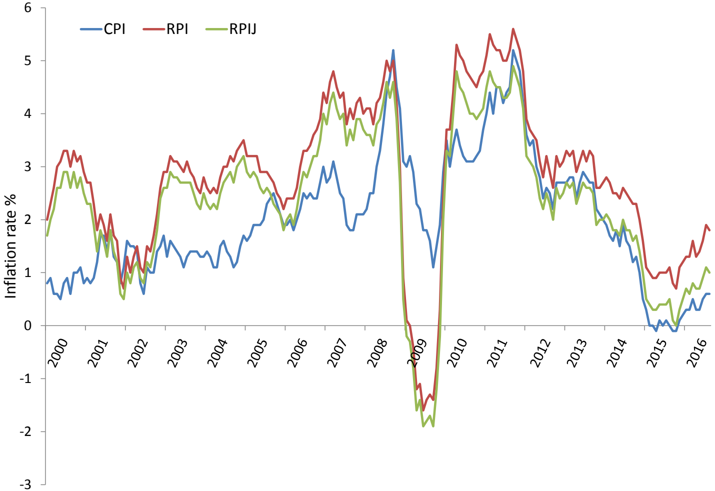
Institute for Fiscal Studies

November 10th 2016

Introduction

- Historically, the UK has had two main measures of consumer price inflation: the CPI, and the RPI
- In March 2013, the ONS started to publish a new inflation index - the RPIJ
- UK Statistics Authority announced it would no longer recognise the RPI as a 'national statistic'
- However the RPI is still published and used
  - uprating excise duties
  - government gilts
  - price caps of regulated industries etc.

RPI, CPI, RPIJ (2000-2016)



RPI and CPI

- Two primary differences
  - Coverage: RPI includes owner occupied housing costs
  - Formula effect: RPI and CPI use different formulae at ‘elementary level’
- In particular, the CPI does not use the *Carli* index
- The RPIJ replaces the Carli index with the Jevons as is used in the CPI
  - Follows recommendation of Diewert (2012)

Is the Carli index flawed?

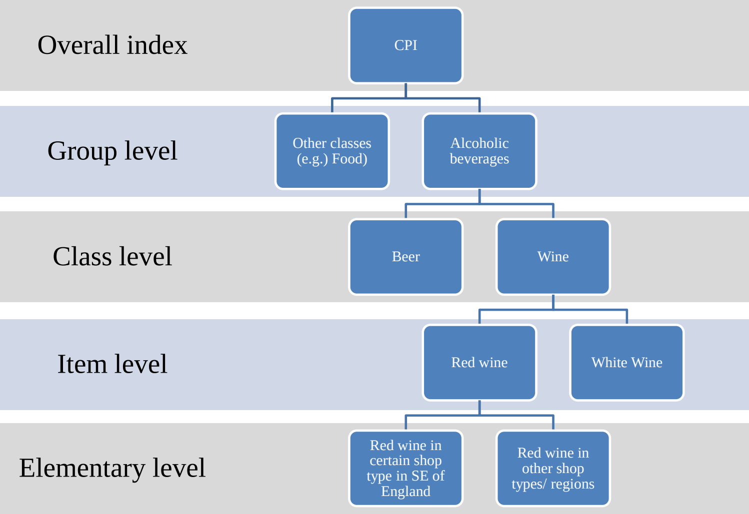
- Discuss and contribute to the three approaches to selecting the appropriate index numbers

1. The *test* approach
2. The *economic* approach
3. The *statistical* approach

- No consensus on which of these is most important!
- Find that none of these offers clear support for the Carli index

Some preliminaries

Aggregation (stylised example)



Aggregation: what is an elementary aggregate?

- While at most stages the ingredients of each index are weighted according to relative quantities purchased...
  - For example we might use the relative budget shares of white wine and red wine
- ....at the elementary level, the ONS *does not have expenditure information*
  - It is much harder to know how much is spent on particular brands of red wine relative to others
- Elementary indices must therefore be unweighted averages of various kinds

Elementary indices

1. The Carli index: $P_C\left(\mathbf{p}_0,\mathbf{p}_1\right)=\frac{1}{N}\sum_{i=1}^{N}\left(\frac{p_1^i}{p_0^i}\right)$
2. The Dutot index: $P_D\left(\mathbf{p}_0,\mathbf{p}_1\right)=\frac{\frac{1}{N}\sum p_1^i}{\frac{1}{N}\sum p_0^i}$
3. The Jevons index: $P_J\left(\mathbf{p}_0,\mathbf{p}_1\right)=\prod_{i=1}^{N}\left(\frac{p_1^i}{p_0^i}\right)^{\frac{1}{N}}=\frac{\prod_{i=1}^{N}(p_1^i)^{\frac{1}{N}}}{\prod_{i=1}^{N}(p_0^i)^{\frac{1}{N}}}$

- Important result: The Jevons is *always* less than or equal to the Carli (Jensen's inequality)

The test approach

The test approach

- Does the index satisfy common-sense axioms?

1. *Positivity:* $P(\mathbf{p}_0, \mathbf{p}_1) > 0$
2. *Identity:* $P(\mathbf{p}, \mathbf{p}) = 1$
3. ...

Time reversal

- The Carli fails some important properties

$$\textit{Time reversal:}\ P\left(\mathbf{p}_0, \mathbf{p}_1\right) P\left(\mathbf{p}_1, \mathbf{p}_0\right) = 1$$

- In fact: $P_C\left(\mathbf{p}_0, \mathbf{p}_1\right) P_C\left(\mathbf{p}_1, \mathbf{p}_0\right) \geq 1$
- Thus the Carli is 'biased' upwards

'Price bouncing' example

Prices	Period 0	Period 1	Period 2	
Shop A	1	1.25	1	
Shop B	1.25	1	1.25	
		Period (0,1)	Period (1,2)	Chained
Carli	...	$(1.25 + 0.8)/2 = 1.025$	1.025	1.0506
Dutot	...	$1.125/1.125 = 1$	1	1
Jevons	...	$\sqrt{1.25 \times 0.8} = 1$	1	1

The test approach: a summary

- The Carli fails the important time reversal test with an upward bias
  - As a result also fails a stronger price bouncing test
- This failure underlay recommendation of Diewert (2012) to end the use of the Carli index in the RPI

How serious is the failure of time reversal?

- Even in an index using the Jevons at an elementary level, the overall index is not time-reversible
  - however it will be *less* sensitive to biases than those constructed using a non-time reversible index

The economic approach

The economic approach

- Does the index do a good job of approximating the cost of living?
  - How much must we increase consumers' incomes to fully compensate them for a price change?
- This approach ideally would explicitly incorporate consumers' substitution responses in response to price changes
  - if price of one good rises relatively to another, consumers might take advantage by substituting away from that good
- Measured by a Cost of Living Index (COLI)

The economic approach

- Two possible COLIs
  - *Laspeyres:* $P_L(\mathbf{p}_0, \mathbf{p}_1) = \frac{\sum p_{\mathbf{1}}^i q_{\mathbf{0}}^i}{\sum p_{\mathbf{0}}^i q_{\mathbf{0}}^i} = \sum w_0^i \left(\frac{p_{\mathbf{1}}^i}{p_{\mathbf{0}}^i}\right)$
  - *Geometric Laspeyres:* $P_{GL}(\mathbf{p}_0, \mathbf{p}_1) = \prod_{i=1}^{N} \left(\frac{p_{\mathbf{1}}^i}{p_{\mathbf{0}}^i}\right)^{w_{\mathbf{0}}^i}$
- where w_0^i corresponds to the budget share of good i in period 0
- Correspond to Leontief ("no substitution") and Cobb-Douglas ("constant shares") preferences respectively
- Not unreasonable to suppose some substitution at the elementary level

The economic approach at the elementary level (1)

- Diewert (2012) writes

  '...the economic approach cannot be applied at the elementary level unless price and quantity information are both available'

- Without budget share information we cannot say that
  - the Carli approximates the Laspeyres or that
  - the Jevons approximates the Geometric Laspeyres
- $\Longrightarrow$ we should use the test or statistical approaches at this level instead

The economic approach at the elementary level (2)

- However, if we want to apply the economic approach consistently at different levels of aggregation then this is not a very satisfying solution!
- Propose instead a constructive principle for selecting appropriate index numbers without quantity information...

Principle of maximum entropy (1)

- Imagine we have a weighted die with a known number of sides but where the probabilities of dice rolls are unknown
  - What can we say about the probability of each individual role?
- Laplace's 'principle of insufficient reason' provides a first step
  - you should set the probabilities to be equal unless you have reason to do otherwise

Principle of maximum entropy (2)

- The PME (Jaynes, 1957a,b) combines Laplace's principle with any information we do have to obtain a prior probability distribution
- An objective function that achieves this is Shannon's entropy function

$$H(\mathbf{p}) = -\mathbf{p}\ln(\mathbf{p})$$

- Maximise this subject to what we do know (e.g. the average dice roll)

PME favours the Jevons (1)

- Apply the PME to vectors of budget shares in both periods

$$\max_{\mathbf{w_0},\mathbf{w_1}} H(\mathbf{w}_0, \mathbf{w_1}) = -\sum_t \mathbf{w_t}' \ln \mathbf{w_t} \text{ subject to } \sum_i w_t^i = 1 \text{ for } t = 0, 1$$

- This will set shares equal across goods in both current and base periods: $w_t^i = 1/N$ for all i and $t = 0, 1$
- Implies both that the Geometric Laspeyres is the target index and that the Jevons approximates this
  - $\Longrightarrow$ favours the Jevons index

PME favours the Jevons (2)

- An additional restriction we can impose is that consumers decisions are governed by restrictions of rational choice
- These are represented by the Generalised Axiom of Revealed Preference (GARP)

$$\max_{\mathbf{w}_0,\mathbf{w}_1} -\sum_t \mathbf{w}_t' \ln \mathbf{w}_t$$

subject to $\{\mathbf{p}_0, \mathbf{p}_1; \mathbf{w}_0, \mathbf{w}_1\}$ satisfies GARP and $\sum_i w_t^i = 1$ for $t = 0, 1$

- It turns out that the solution to this problem is the same: $w_t^i = 1/N$ for all i and $t = 0, 1$

Summing up

- Statistical approach (not discussed) is context dependent
  - Carli and Jevons differ in bias and variance properties
- Test approach favours the Jevons over the Carli
- Economic approach favours the Jevons over the Carli
- Hence I concur with the decisions of ONS and United Kingdom Statistics Authority

References

- Diewert, W. E. (2012), "Consumer Price Statistics in the UK", Office for National Statistics.
- Jaynes, E. T. (1957a) ,"Information Theory and Statistical Mechanics", *Physical Review*, 106, 620-630.
- Jaynes, E. T. (1957b), "Information Theory and Statistical Mechanics II", *Physical Review*, 108, 171 - 190.