Comments on "Testing by Betting" by Glenn Shafer

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Covid-19 and Hydroxychloroquine

Currently there are … randomized clinical trials underway to investigate whether Trump's Miracle Drug helps curing COVID-19

Any guess how many?

Covid-19 and Hydroxychloroquine

Currently there are **156** randomized clinical trials underway to investigate whether Trump's Miracle Drug helps curing COVID-19

Some of these will be significant, some won't. Many of these have been started because previous ones gave hopeful interim results. How to combine results?

Use E-values (Betting Scores), not p-values!

Avoiding Research Waste with ALL-IN Meta-Analysis - joint work with Judith ter Schure

Betting and Type-I Error Control

- I agree 100% with Shafer. Still I would like stress there's more to Betting Scores than communication:
- **Best of Both (likelihoodist/Neymanian) Worlds**: 'evidence as data accumulates', even over several studies $Y_{(1)}, Y_{(2)}$, ... with different alternative distrs.
  - [you simply multiply betting scores]
- Yet – and this might convince practitioners - there is still **Type-I error control** (Ville's Inequality - Shafer & Vovk 2019 – "α-warranty over time"):

$$P_0\left(\exists n : \prod_{i=1}^{n} S(Y_{(n)}) \geq \frac{1}{\alpha}\right) \leq \frac{1}{\alpha}$$

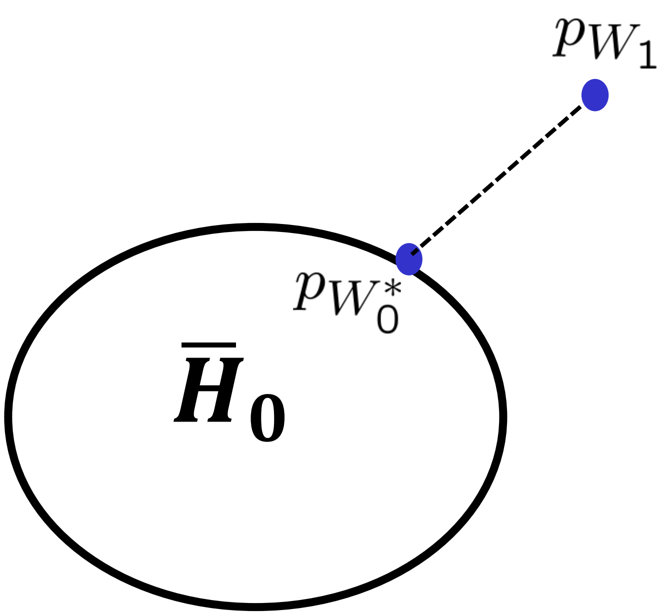
Betting and Bayes

- There's more to Betting Scores than communication:
- **[Best of Three Worlds?]** For a simple null hypothesis, every Bayes factor is also a betting score
- For composite null hypotheses, most Bayes factors are **not not not** betting scores in Shafer's sense.
  YET for every prior W_1 on alternative H_1, there is "matching" prior W_0^* on H_0 such that the resulting Bayes factor is a valid (and W_1-optimal) betting score!
  - *Safe Testing (with R. de Heide W. Koolen, A. Ly R. Turner and M. Perez)*

Reverse Information Projection gives Bayesian W_1-optimal bets

$$p_W(Y^n) := \int p_\theta(Y^n) dW(\theta)$$

$$W_0^* := \arg \min_{W_0 : \text{distr on } \Theta_0} D(P_{W_1} \| P_{W_0})$$



Thm (G. Koolen, De Heide, *Safe Testing*, 2019):

For every prior W_1 on Θ_1,

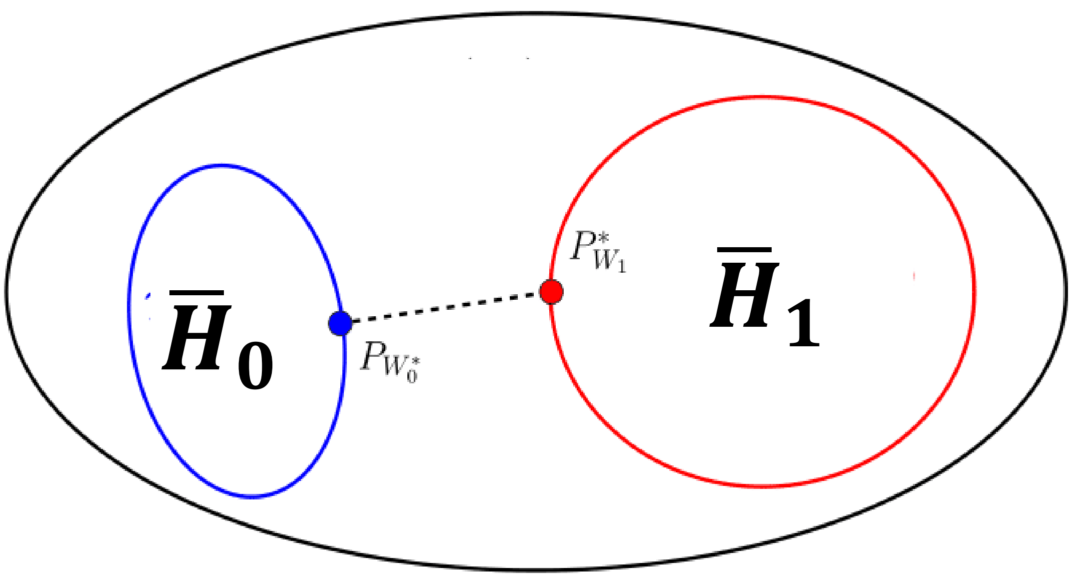
$$S := \frac{p_{W_1}(Y^n)}{p_{W_0^*}(Y^n)}$$

is a valid betting score, and it is the GROW-(log)-optimal one

For separated H_0 and H_1, best betting scores given by Joint Information Projection (JIPr)

$$p_W(Y^n) := \int p_\theta(Y^n) dW(\theta)$$

$$(W_1^*, W_0^*) := \arg \min_{W_1 \text{distr on } \Theta_1} \min_{W_0 \text{:distr on } \Theta_0} D(P_{W_1} \| P_{W_0})$$



Resulting betting scores $S := \frac{p_{W_1^*}(Y^n)}{p_{W_0^*}(Y^n)}$ often grow much faster (provide more evidence) than those achieved by calibrating p-values!

For separated H_0 and H_1, best betting scores given by Joint Information Projection (JIPr)

$p_W(Y^n) := \int p_\theta(Y^n) dW(\theta)$

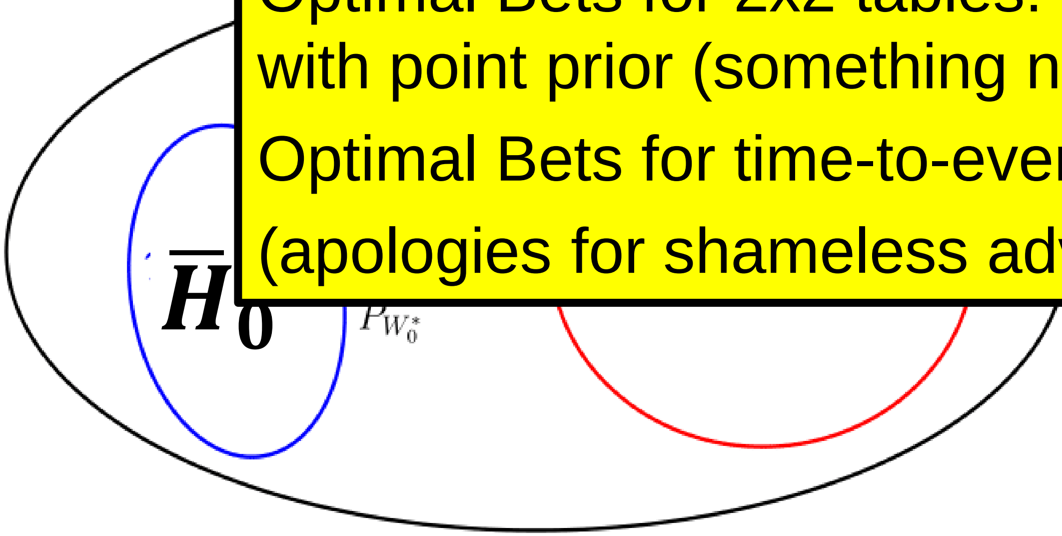
$(W_1$ … $P_{W_1} \| P_{W_0})$

Optimal Bets for testing mean of a normal: Bayes factor with right haar prior on variance (Bayesian t-test)

Optimal Bets for 2x2 tables: Bayes factor with point prior (something new)

Optimal Bets for time-to-event-data…

(apologies for shameless advertisement)



… betting

$$\frac{p_{W_1^*}(Y^n)}{p_{W_0^*}(Y^n)}$$

… much faster (provide more evidence) than those achieved by calibrating p-values!